

Eg. 1) $\{ (3, 3, -3), (6, 6, -6) \}$
 2) $\{ (3, 3, -3), (6, 6, -6), (2, 3, 1) \}$

Result: superset of L.D. set is L.D.

Eg. 2) $S = \{ (1, 0, 0)^{e_1}, (0, 1, 0)^{e_2}, (0, 0, 1)^{e_3} \}$

$$(x, y, z) = x(1, 0, 0) + y(0, 1, 0) + z(0, 0, 1)$$

$$S_1 = \{ (1, 0, 0), (0, 1, 0) \} \text{ L.I.}$$

Result: subset of L.I. set is also L.I.

Def: For $\alpha_1, \alpha_2, \dots, \alpha_n \in \mathbb{R}$, $x_1, x_2, \dots, x_n \in V$ if
 $\alpha_1 x_1 + \alpha_2 x_2 + \dots + \alpha_n x_n = 0$

and all $\alpha_i = 0$ then $\{ x_1, x_2, \dots, x_n \}$

is L.I.

Eg: $\{ (2, 3, 1)^{x_1}, (1, -1, 2)^{x_2}, (7, 3, 8)^{x_3} \}$ L.I. or L.D.?

$$\alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 = \vec{0}$$

$$\alpha_1 (2, 3, 1) + \alpha_2 (1, -1, 2) + \alpha_3 (7, 3, 8) = \vec{0}$$

$$(2\alpha_1 + \alpha_2 + 7\alpha_3, 3\alpha_1 - \alpha_2 + 3\alpha_3, \alpha_1 + 2\alpha_2 + 8\alpha_3) = (0, 0, 0)$$

$$2\alpha_1 + \alpha_2 + 7\alpha_3 = 0$$

$$Ax = 0$$

$$3\alpha_1 - \alpha_2 + 3\alpha_3 = 0$$

$$\alpha_1 + 2\alpha_2 + 8\alpha_3 = 0$$

$$\begin{bmatrix} 2 & 1 & 7 & 0 \\ 3 & -1 & 3 & 0 \\ 1 & 2 & 8 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 1 & 7 & 0 \\ 0 & 5 & 15 & 0 \\ 0 & 3 & 9 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 1 & 7 & 0 \\ 0 & 5 & 15 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A|h) = 2 < n(3)$ Infinitely many sol.

$\rho(A|b) = 2 < n(3)$ Infinitely many sol.

Then the free variable can be chosen non zero & hence the sol is L.D.

Method 2¹

$$\begin{bmatrix} 2 & 3 & 1 \\ 1 & -1 & 2 \\ 7 & 3 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 1 \\ 7 & 3 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 2 \\ 0 & 5 & -3 \\ 0 & 10 & -6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 5 & -3 \\ 0 & 0 & 0 \end{bmatrix}$$

Eg: $\{ \overset{e_1}{(1, 0, 0)}, \overset{e_2}{(0, 1, 0)}, \overset{e_3}{(0, 0, 1)}, (1, 2, 3) \}$ L.I.

A.L.I. set for $\mathbb{R}^4$.

$\{ \overset{e_1}{(1, 0, 0, 0)}, \overset{e_2}{(0, 1, 0, 0)}, \overset{e_4}{(0, 0, 0, 1)}, \overset{e_3}{(0, 0, 1, 0)} \}$ L.I.

Basis: B i) B is L.I. ii) $\text{Span}(B) = V$

Dimension: no. of elements in the basis set.

Eg: H.W. $X = \{ (1, 2, 0), (0, 3, 1), (-1, 0, 1) \}$ L.I. or L.D.

L.I. $\dim X = 3$